647510 **BSc5MatC503x** **Seat No :________**

B.Sc. Semester - 5 (*CBCS*) Examination
Oct/Nov - 2022 (NEW COURSE)
Boolean Algebra & Complex Analysis -1(Core)

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1(A) Answer any two: **(10)**

(1) Define: Equivalence relation. Show that the relation congruence modulo m, in the set Z of integers is an equivalence relation.

(2) If (L, ≤)is a lattice and $a, b, c \in L$ then prove that

(i) $a \oplus (b * c) \leq (a \oplus b) * (a \oplus c)$

(ii) $a * (b \oplus c) \geq (a * b) \oplus (a * c)$

(3) Prove that the direct product of two lattices is also a lattice.

(B) Answer any one: **(04)**

(1) Define: Partial order set. Let Z be the set of integers. Define $R = \{(a, b)/\ b = a^r, for\ some\ r \in N, a, b \in Z\ \}$, show that (Z,R) is a poset.

(2) Let A be a non-empty set and P(A) be the power set of A. For $X, Y \in P(A), X * Y = X \cap Y, X \oplus Y = X \cup Y$. Prove that $(L,*,\oplus)$is a lattice.

Q.2(A) Answer any two: **(10)**

(1) State and prove unique representation theorem of Boolean algebra.

(2) If $(B,*,\oplus,', 0 , 1)$ is Boolean algebra, then for any $x_1, x_2 \in B$, Prove that :

(i) $A(x_1 * x_2) = A(x_1) \cap A(x_2)$

(ii) $A(x_1 \oplus x_2) = A(x_1) \cup A(x_2)$

(3) Define: Boolean function, Find cube array representation of the Boolean function $f(x, y, z) = x'y + xy$

(B) Answer any one: **(04)**

(1) Express the Boolean expression $\alpha(x_1, x_2, x_3) = x_1 + x_2 + x_3$ as the sum of product canonical form.

(2) If $(B,*,\oplus,', 0 , 1)$ is Boolean algebra then prove that:

(i) $(a * b)\oplus(b * c)\oplus(c * a) = (a\oplus b) * (b\oplus c) * (c\oplus a)$

(ii) $(a * b)' \oplus (a \oplus b) = 1$

Q.3(A) Answer any two: **(10)**

(1) Show that Cauchy-Riemann conditions are sufficient for a complex function f(z) to be analytic.

(2) If $f(z) = u + iv$ is analytic function, where u and v are harmonic functions of r and θ satisfies Cauchy-Riemann conditions then prove that: $f'(z) = (cos\theta - isin\theta)\left(\frac{\partial u}{\partial r} + i\frac{\partial v}{\partial r}\right)$.

(3) Define: Harmonic Function. If u and v are conjugate harmonic functions then prove that the family of curves obtained by $u = c_1$ and $v = c_2$ are orthogonal.

(B) Answer any one: **(04)**

(1) Find an analytic function $f(z)$ such that $Re[f'(z)] = 3x^2 - 4y - 3y^2andf(1 + i) = 0$.

(2) Prove that $u = logr$ is harmonic and find its conjugate.

Q.4(A) Answer any two: **(10)**

(1) State and prove Cauchy's integral formula.

(2) If C is the circle $|z| = R, R > 1$ described in the counter clockwise direction then show that $\left|\int_c \frac{logz}{z^2} dz\right| < 2\pi\left(\frac{\pi+logR}{R}\right)$

(3) Define: Closed curve, prove that $\int_c \pi e^{\pi\bar{z}}\, dz = 4(e^{\pi} - 1)$, where c is the square joining points $z = 0, z = 1, z = 1 + i$ and $z = i$.

(B) Answer any one: **(04)**

(1) Evaluate $\int_c z^2\, dz$ along the straight line from 0 to 1 + I.

(2) Find $\int_c \frac{zdz}{(9-z^2)(z+i)}$, C : $|z|$=2

Q.5(A) Answer any two: **(10)**

(1) State and prove Cauchy's Integral formula for derivative and deduce that $f^n(z_0) = \frac{n!}{2\pi i}\int_c \frac{f(z)}{(z-z_0)^{n+1}} dz.$

(2) State and prove Morera's theorem.

(3) Prove that $\int_c \frac{e^{kz}}{z} dz = 2\pi i, c: |z| = 1$ and hence deduce that

(i) $\int_0^\pi e^{kcos\theta} \cos(ksin\theta)\, d\theta = \pi$ (ii) $\int_{-\pi}^{\pi} e^{kcos\theta} \sin(ksin\theta)\, d\theta = 0.$

(B) Answer any one: **(04)**

(1) Evaluate$\int_c \frac{5z^2-7z+3}{(z-1)^2} dz, C: |z| = 2$

(2) Evaluate$\int_c \frac{z^3+2}{(z+i)^3} dz, C: |z| = 2$

****